

A Binary-tree Based Set Partition Modulation Optimization

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Abstract

From Permutation Modulation to Index Modulation, works have been done to find an effective way to distribute signals into certain regions. In this paper, an improved set partition modulation (SPM) scheme is advanced. We take product-distance into consideration on basis of previous work. Binary-tree method is introduced into the selection of code-words for better product-distances. The exact best code-book can be found without complexity in measures as brute force.

Keywords

Index Modulation; OFDM; Code-book.

1. Introduction

In [1], a scheme named Permutation Modulation(PM) was first represented to partition the signal space into disjoint regions. Yet, as 5G wireless network technology ascending, [2] have described a more promising index modulation (IM) technique based on PM. It could be realized by spatial modulation (SM) [3] in multi-antenna communication or sub-carrier activation when it comes to OFDM transmission [4]. In order to gain a better performance in OFDM occasions, multiple IM-based schemes have been considered. Improvements mainly focus on the state of the sub-carriers. For instance, in dual mode OFDM with IM (DM-OFDM-IM) [5] [6], the sub-carriers are divided into several subblocks where sub-carriers are grouped again and modulated respectively by a pair of distinguishable modem-mode constellations. In multi-mode OFDM-IM (MM-OFDM-IM) [7] Both SM-OFDM-IM and MM-OFDM-IM could result in a performance which has less errors. While methods above encode information by permutations or combinations, a new idea called set partition modulation(SPM) is described [8].

The set partitioning algorithm has already been a common math method in researches. For example, it was first applied in code-mapping by Ungerboeck for 8 phase-shift keying and 16 quadrature amplitude-shift keying modulation [9]. The basic concept is that, in a certain set partition, the union of all groups makes the whole set while groups themselves are disjoint. [8] first introduced the method into OFDM transmission. Set partition modulation(SPM) encodes information by mapping a single code-word to a partition of the whole set into subsets and twisting the constellation to maximize the distance between points. However, while the Hamming distance between code-word pairs is considered through [8] as one criteria, the product-distance is ignored due to complexity of the selection problem.

Against this background, we proposed an advanced SPM model. The selection of code-words is based on a binary-tree. In the previous work, as a OFDM-IM criteria, if the Hamming distance between code-words is greater or equal to two, connections of these vertices are made.

Otherwise one of them would be excluded from the code-book. This is to keep similar Hamming distance properties and gain diversity. However, in the latter case, canceling which code-word and how the choice may affect the performance were not discussed. There is an analogy between listing all possible situation and naming a binary-tree, since each time we only have two code-words to choose. After building the tree, product distance comparison from the bottom could be done between all the way up to the root in every branch. By using this method, it is efficient and clear to find the best solution of code-word selection.

The rest of this report is organized as follow. In Section II, we will review the basic idea of SPM. Details on the selection methodology will be explained in Section III.

2. System model and assumptions

In our model, we use a basic idea of SPM. These ideas were first published and some definitions and relations were taken by Yarkin in [8]. We will briefly explain some of them as follows.

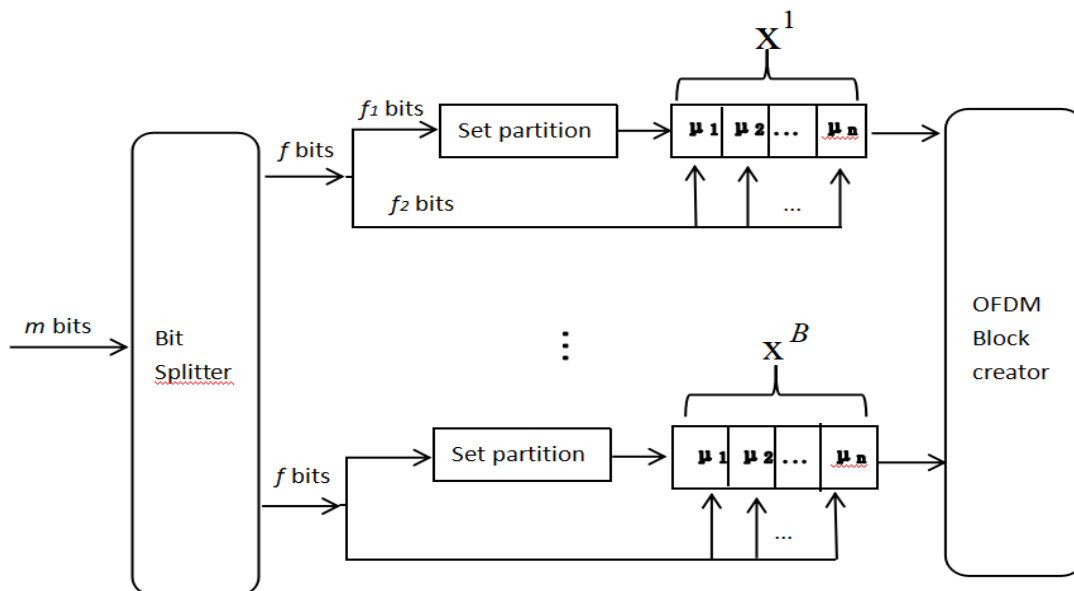


Fig. 1 Transmitter structure of the OFDM-SPM scheme.

SPM system can be divided as SPM, Full SPM(FSPM), Ordered SPM(OSPM) and Ordered Full SPM(OFSPM). In a full SPM system, a code-book which has L code-words $\mathbf{x}_1, \mathbf{x}_2, \dots, \mathbf{x}_L$ are considered to partition an N -element set χ into non-empty subsets for every possible way. The code-book are came from constellation diagram which is an M -ray symbol and each code-word has N elements, i. e., $\mathbf{x}_i = \{x_{i1}, x_{i2}, \dots, x_{iN}\}, i = 1, 2, \dots, L$. In this system, it has all possible set partitions which are unordered. But in the simply SPM system which has the same code-book, each code-word is partitioned into K subsets as K is a certain number while in FSPM $K \in \{1, 2, \dots, N\}$. The code-book size L is the number of ways to set partition, and $L_{FSPM} = \sum_{K=1}^N L_{SPM}$. As what have illustrate above, the SPM system is an ordinary one which has less set partitions. We will use the SPM system to construct our system model.

We present a system model which apply a SPM scheme to OFDM transmissions. The transmitter structure of the OFDM-SPM scheme is shown in Fig. 1. In the given scheme, m input bits are divided into $B = m/f$ blocks in the bit splitter, as each block has f input bits during the SPM transmitter. Similarly, N_T sub-carriers are also divided into $B = N_T/N$ blocks, as each block has N sub-carriers. For each block of f input bits, they are modulated by an SPM encoder and the resultant modulated symbols are carried by N sub-carriers.

As each bit and each sub-carrier block have the same mapping operation and the SPM system is the base of other three system (FSPM, OSPM, OFSPM), we can only focus on one single block and consider the SPM system. In the b th block (where $b \in \{1, 2, \dots, B\}$), the SPM encoder divides f bits into f_1 bits and f_2 bits and $f_1 + f_2 = f$. The f_1 bits are used to determine the set partition of χ ($\chi := \{x_1, x_2, \dots, x_N\}$), we can get LSPM different set partitions which belong to the SPM system defined above. Each set partition is mapped to a SPM code-word x_i^b and each element in the SPM code-word corresponds to a sub-carrier in the b_{th} block. The other f_2 bits are used to modulate symbols on the N sub-carriers which correspond to the set partitions of the f_1 bits defined above. We use different subsets to define specific SPM code-words, since the SPM code-words are modulated on each sub-carrier, we can assume that each subset identifiers μ_k is an element of a M -ray constellation M_k , (where $\mu_k \in M_k, k \in \{1, 2, \dots, N\}$). We can assume each constellation as M , therefore $f_2 = N \log_2 M$.

As what have shown in [7], we can get constellations M_k by rotating each constellation with the angle of $(k-1)\pi/N, k \in \{1, 2, \dots, N\}$. The f_1 bits are mapped to different set partitions, but for a certain number of sub-carriers and subsets, we only need 2^{f_1} set partitions. Therefore, code-book selection become a important part as what we discuss in the next section.

After all blocks have transmitted the SPM code-words along with the modulation symbols, an OFDM block creator forms these symbol vectors and decoding them. Since we mostly focus on the modulation and code-book selection, we do not make the decoding part in our model.

3. Code-book selection methodology

The BER performance of a fading channel will be limited by the minimum Hamming distance at a high signal-to-noise(SNR).

$$d_{min} = ||X^i - X^j|| \quad (1)$$

Note that $X^i = \text{diag}(x^i)$, $X^j = \text{diag}(x^j)$ and $(X^i - X^j)$ corresponds to the minimum rank among all X^i and X^j pairs.

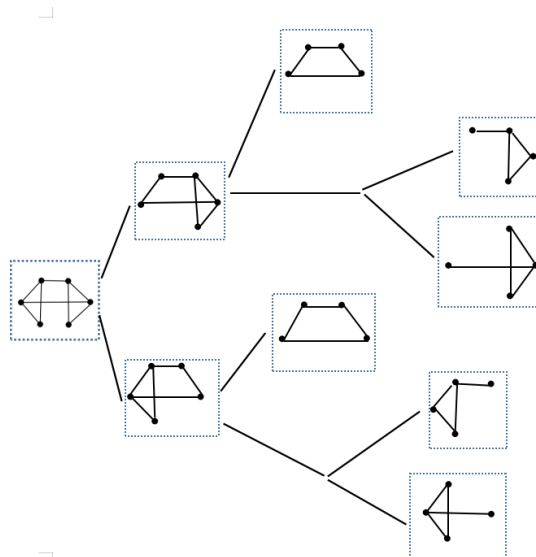


Fig. 2 Possible code-book with same diversity binary tree

For the set partition, we can get a code-words and the number of the code-words might not equal to the power of 2, therefore we need to choose the code-book. It will be a difficult to consider all possible cases. For example if we have 3 sub-carriers and use BPSK, there will be 24 code-words in SPM, and there will be 735,471 possible cases to choose 24 code-words into

16 code-words. Therefore, we can regard the selection problem as a maximum clique problem or k -clique problem in a graph. In the [8] due to the complexity of the code-book selection problem, the selection algorithms do not consider product-distances. Here we will consider the product-distances to optimize the chosen code-book in order to get better BER performance.

In a small system, we use the brute force to find the best code-book which will be used as the benchmark of our algorithm. The brute force algorithm has been declared in [8] clearly here we do not repeat it again. Hence, it will be NP-complete in large system to select the best code-book, we propose an algorithm to find sub-optimal code-book by excluding the vertices which has minimum number of edge. This algorithm will consider both the hamming distances and product-distances. The algorithm will base on the the algorithm 3 in [8] and add binary tree to consider the product-distances to get better sub-optimal code-book.

To make it easier to explain, we begin with some definitions. In undirected graph $G(V, E)$ where $V = \{v_1, v_2, \dots, v_L\}$ is the vertex set and E is the edge set of G . The $N(v_i)$ denotes the cardinality of the neighborhood of each node where v_i in V . r denotes the binary tree $r(k)$ denotes the root of the binary tree, $r_r(k)$ denotes the right branch of the root $r(k)$ and $r_l(k)$ denotes the left branch of the root $r_r(k)$. At the beginning of the algorithm, the graph is initialized with the hamming distance. Where $V = \{v_1, v_2, \dots, v_L\}$ will be the set of SPM code-words and the edge in E denote the Hamming distance not less than 2 between v_i and v_j for any v_i and v_j in V where v_i is not equal to v_j . And then calculate the cardinality of neighborhood $N(v_i)$ for each node. Then find a clique by excluding vertices. Do $v_{min} = \arg \min_{v_i \in V} N(v_i)$, $G = G - \{v_{min}\}$, $V = V - \{v_{min}\}$, calculate new $N(v_i)$, $N(v_i) = N(v_i)$ while $N(v_i)$ is not equal to $V-1$. And when excluding the vertex, if there are other vertices has equal number of neighborhood, put the different choosing case into binary tree r to be the $r_l(k)$ or $r_r(k)$. After finishing the loop above we will get a binary tree r has leaves denotes different code-book with same diversity which is shown in Fig. 2. Finally, we calculate the minimum product-distances of ever leaves in the binary tree r and find the largest one to be the finally chosen code-book.

4. Results

After the choosing by considering both Hamming distances and product-distances, the data rate will not change from choosing by only considering the Hamming distances, and the data rate has been analysed in [8], here we do not analyse it again, we just compare the BER performance of the code-book chosen by brute force, average BER performance of five code-books chosen by Hamming distances only and BER performance of the code-book chosen by both Hamming distances and product-distances.

5. Conclusions

Using brute force to select code-books in more set parts condition is a NP-hard problem. In previous paper of SPM, Jutstin Coon used a method of Graph theory selected the code-books only based on Hamming distance. This paper using binary tree to record every step of selecting code-books using method of Graph theory and then use traversal of the binary tree to select code-books which has better product-distances and performance in among the code-books having the same Hamming distances.

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